Menofia University Faculty of Engineering Shebien El-kom Basic Engineering Sci. Department. Academic Year: 2017-2018 Date: 30 / 12 / 2017	Subject: Mathematical Models Code: BES 510 Time Allowed: 3 hours Year: Master Total Marks: 100 Marks

Answer all the following questions: [100 Marks]

Q.1 **(A)** ***Write brief notes on the following topics:*** [40]

(1) The mathematical models? And What are the benefits of using the mathematical models?

(2) The characteristics of the mathematical model?

(3) The types of mathematical models?

(4) The steps of constructing a mathematical model?

(5) The computational modeling? and state some examples of computational modeling? And how can computational modeling improve medical care and/or biomedical research?

(6) The physical Modeling? The aims of physical modeling?

(7) Draw the flow chart for relation between physical and Mathematical Model?

(8) State the classes of physical modeling?

(9) Define Dynamic System and Mention some examples to illustrate it.

(10) State the evolution rule of dynamic system.

(11) Define the Probabilistic Model? and where do the uncertainty occur? And state some Examples on Probabilistic Model.

(12) Explain chaos theory?

(B) Write a mathematical model to describe the motion of a block with mass (m) and is connected to a vertical spring of spring constant (k). A block stretches the spring by a distance xs from its unstretched position when the system is in equilibrium as in the figure below. Determine the mass of the object, the maximum amplitude of oscillation such that the top block will not slip on the bottom block.

(C) Write a mathematical model to describe the motion of a projectile of mass $(m)\ kg$ and diameter $(d)\ m$ with launching speed (V_P) with launching angle (α) using the evolution rule to determine the position of the particle and draw the trajectory of the particle.

(D) Find and Solve the Difference equation associated with the following sequence $7.17.37.77.157$.

Q.2 **(A)** Consider the following dynamical system $a_{n+1} = \frac{1}{1+b_n}; b_{n+1} = \frac{1}{4+a_n}$. [30]

Find its steady states and discuss its behavior for any positive value of initial values.

(B) The model $y = y_o\ e^{kt}$, with $k > 0$, is sometimes used to model bacterial growth.

(i) Describe the qualitative predictions made by the model. In particular, show that

$$G(t) = \frac{y(t+1)}{y(t)},$$ does not actually depend on t.

(ii) Describe an experiment that tests the prediction of part (i).

(iii) Describe a physical setting in which this model for population growth is clearly not appropriate.

(iv) Describe a physical setting in which this model for population growth might be appropriate.

(C) One of the data sets in below table has the origin as its mean point.

(i) Find the equation of the straight line that best fits that data.

(ii) Plot the data and the best-fit line together on a graph.

x	-4	-1	0	2	3
y_1	-5	-2	0	2	4
y_2	-5	-2	1	2	4

(D) Prove that $B(X_1.\cdots.X_n;Y)$ is a Banach space by using the natural identification $B(X_1.\cdots.X_n;Y) \cong B\big(X_1.\ B(X_2.\cdots.X_n;Y)\big)$.

Q.3 **(A)** Derive the results of the linear least squares method for the model [30]

$y = mx$,

$$m = \frac{\sum xy}{\sum x^2}, \quad RSS = \sum y^2 - \frac{(\sum xy)^2}{\sum x^2} = \sum y^2 - m\sum xy,$$

by applying optimization methods from calculus to the total discrepancy function: $F(m) = (\sum x^2)\, m^2 - 2\,(\sum xy)\, m + (\sum y^2)$.

(B) Two possible models for the dynamics of a renewable resource (biotic or abiotic) are

$$\frac{dx}{dt} = 0.1 - \frac{xy}{1+x} \quad \text{and} \quad \frac{dx}{dt} = 0.1\,x - \frac{xy}{1+x}$$

where $x(t)$ is the amount of resource present at time t and y is the number of consumers.

(i) For each of these models, describe a mechanism that accounts for the growth of the resource in a way that is consistent with the model.

(ii) Explain the assumption the models make about the consumers.

(C) Solve the following 2nd order difference equation:

$$x_{n+2} - 2\,x_{n+1} + 2\,x_n = 0$$

with initial conditions $x_1 = 0, x_2 = 1$.

(D) Solve the following system using Euler's method

$$x' = -2tx + 3y^2, \quad y' = -3x^2(1-y)$$

with I.C $x(0) = -1$, $y(0) = 2$. With step size $h = 0.1$, from $t = 0.1$ *to* $t = 0.5$

This exam measures the following ILOs								
Question Number	Q1-a	Q1-b	Q3-b	Q4-a	Q1-c	Q2-a	Q3-a	Q4-c
	Q4-b				Q2-b	Q2-c	Q3-c	
	Knowledge &understanding skills				Intellectual Skills		Professional Skills	

Good Luck

Dr. Ramzy M. Abumandour